Restraint of purlins for various roof systems

T. Vrany¹, M. Braham² & A. Belica²
¹Faculty of Civil Engineering, Czech Technical University, Praha, Czechia
²Astron Buildings S.A., Diekirch, Luxembourg, A member of the Lindab Group

ABSTRACT: In the analysis of the cold formed purlins the stabilising effect of the connected cold formed sheeting must be inevitably considered. This effect is represented by the shear and rotational restraints. Extensive experimental and theoretical research was carried out to determine the rotational and lateral restraints of the purlins in Astron roof systems. In addition to the typical roofs with the sheeting screwed down directly to the purlins, the spacer systems with brackets and rails and the standing seam panels with the friction free thermal expansion joints were analysed. Double skin roofs were studied as well. Due to the presence of the spacer elements in those systems and due to the discrete connections between the purlin and the sheeting, their behaviour is more complicated than the behaviour of the traditional roofs. The methodology of the analysis is presented. The method of components is applied, the appropriate stiffness of individual roof components is determined using either experiments or analytical solution. General method analysing the effect of the purlin load on the rotational restraints is presented. It is based on the study of the contact forces between the purlins and roof elements and is verified by experiments.

1 INTRODUCTION

This paper refers to the extensive research project focused on the determination of the rotational and shear restraints for Astron roofs and walls with purlins. Project has been carried out in several waves. The roof systems were tested in 2007 and 2008, the wall system tests are ongoing. This paper presents results of the roof systems and is focused on the most interesting and so far not published topics. Two stiffness parameters, namely the rotational stiffness \( C_D \) and the shear stiffness \( S \) were investigated for each roof type.

Rotational stiffness \( C_D \) is defined as a torsion moment related to the top flange of the purlin developing the unit rotation of a flange. Its unit is Nm/m/rad.

Shear stiffness \( S \) is defined as a lateral force developing the unit lateral displacement of a purlin unit length. Its unit is N/m/m.

2 EXAMINED ROOF TYPES

Large variety of the examined roof systems were grouped according to similarity of their behaviour into four categories:

RT1: Roof type 1 is the basic roof with the corrugated sheeting screwed down to the purlin directly or through the insulation from mineral wool and/or the polyurethane isoblock, see Figure 1a).

RT2: Roof type 2 is the roof with the standing seam sheeting having thermal expansion clips screwed down to the purlin, see Figure 1b).
RT3: Roof type 3 is the double skin roof with the spacer system, omega shaped spacers and rails. Inner skin is done from the corrugated sheeting and outer skin either from the corrugated sheeting (RT3a, see Figure 1e), or from the standing seam sheeting with thermal expansion clips (RT3b).

RT4: Roof type 4 is the single skin roof, with the spacer system containing bridge brackets and bridge rails, having the outside skin either from the corrugated sheeting (RT4a), or from the standing seam sheeting with the thermal expansion clips (RT4b), see Figure 1c, d).

Figure 1. Analysed roof types.

3 ROTATIONAL RESTRAINT

3.1 Scope of work, tests

In order to determine the rotational restraints of the various roof systems 119 tests using 74 different specimens were done. Several specimens were used 2 times to investigate the rotational restraints in two opposite directions of the lateral loading. Testing procedure complied with the EN 1993-1-3 provisions. Loads were introduced by the loading jack or by the weights. Examples of test specimens are shown in Figure 2 and Figure 3. In addition to the standard tests, 29 tests were done to simulate the influence of the direct loading on the purlin. These tests are described in paragraph 3.3.
3.2 Methodology of analysis

The rotational restraint $C_D$ for each RT1 test can be determined from the equation (10.18) of EN 1993-1-3 and the expression $K_B$ for the purlin stiffness is modified for cases, when the deflection measurement point or the load application point is not in the plane of the free flange:

$$K_B = \frac{E t^3 l_B}{4 \left(1 - \nu^2\right)} \left[ \frac{h_s^2}{2} \left(3h - h_s\right) + b_{\text{mod}} h h_s \right]$$

(1)

where:
- $h_s$ is the distance between the deflection measurement point and inner face of the sheeting and
- $h$ is the distance between the load application point and inner face of the sheeting.
The statistical evaluation was done for each roof variant. Due to the relatively large scatter of results, the tests were grouped. Following procedure was applied for each group having the same sheeting and insulation, but the different purlin type. The rotational flexibility \(1/C_D\) consists of the local flexibility of the top flange of the purlin \(\phi_b\) and the local flexibility of the sheeting \(\phi_s\) (involving also the effect of the insulation):

\[
C_D^{-1} = \phi_b + \phi_s \tag{2}
\]

The flexibility \(\phi_s\) for the specific purlin can be found using the method described in Vraný (2002). Its principle is illustrated on Figure 4. The rotational flexibility of the connection is caused by the mutual vertical displacement \(\delta\) of the compression contact line and the line of screws.

![Figure 4. Purlin-sheeting contact area - model of behavior.](image)

From the Figure 4 follows:

\[
C_{Db} = \frac{1}{\phi_b} = K_{Db} a^2 \tag{3}
\]

\[
K_{Db} = \frac{1}{\frac{\delta_b}{p}} \tag{4}
\]

where \(p\) is the pitch of fasteners along the purlin length and \(\delta_b\) is the local deflection of the top flange of the purlin in the point of fastener due to the unit force; using Vraný (2002):

\[
\delta_b = 8.8 \times 10^{-4} \frac{b_a^2}{t_b^3} \text{ [mm/kN]} \tag{5}
\]

\(b_a\) defined in Figure 4 and \(t_b\) is the purlin sheet thickness.

The flexibility \(\phi_s\) is then determined for each test and statistically evaluated for the whole group.

The total stiffness of the spacer roof systems RT3 and RT4 can be determined by the component method. In case of RT4 (single skin roof with brackets and rails) the partial stiffness was determined for different levels. Two procedures were applied:

- lateral deflection was measured at two different depth levels of the purlin, see Figure 3b), to obtain two test values for two unknown rotations,
- partial tests without purlins were executed, see Figure 3a); the flexibility of the purlin-bracket connection was excluded from the results of these tests.

Both, bracket spacers and the thermal expansion clips were considered as a rigid bodies. Then the stiffness at each level of the roof was determined as:

\[
C_{D(AB)} = \frac{M}{\phi} = \frac{F h_F}{\delta/h_A} \tag{6}
\]

where:

- \(h_A\) is the distance between the deflection measurement and inner surface of the sheeting,
- \(h_F\) is the distance between the load application and inner face of the sheeting,
- \(F\) is applied force and
- \(\delta\) is measured lateral deflection.
Distortion of the rail, which depends non-linearly on the mutual positions of the bracket spacers and the thermal expansion clips, was determined by the numerical analysis. Then the final rotational stiffness was calculated as:

\[ C_D = \frac{1}{\frac{\ell_{\text{clip}}}{C_{D(A)}} + \frac{1}{C_{D(B1)}} + \frac{\ell_{\text{bracket}}}{C_{D(B2)}} + \frac{\ell_{\text{bracket}}}{C_{D(C)}}} \]  

(7)

where

- \( C_{D(A)} \) is the rotational stiffness of the thermal expansion clips and sheeting contact,
- \( C_{D(B1)} \) is the rotational stiffness corresponding to the rail distortion,
- \( C_{D(B2)} \) is the rotational stiffness of the thermal expansion clip, rail and the bracket contact,
- \( C_{D(C)} \) is the rotational stiffness of the bracket and purlin contact,
- \( \ell_{\text{clip}} \) is the distance of the thermal expansion clips and
- \( \ell_{\text{bracket}} \) is the distance of the bridge brackets.

For the RT3, the double skin roof according to Figure 1e), the total stiffness is given as a sum of the outer roof system \( C_{D,1} \) stiffness (according to equation (7)) and the inner skin stiffness \( C_{D,2} \):

\[ C_D = C_{D,1} + C_{D,2} \]  

(8)

### 3.3 Effect of purlin load

The torsion moment related to the purlin-sheeting connection is caused by the external purlin load. The ratio of the lateral load producing torsion and the purlin external load, depends exclusively on the cross-section of the purlin. According to Figure 5, \( \tan \alpha = q_\perp / q = k_q \).

In order to determine the effect of the purlin external load on the rotational stiffness \( C_D \), the original test arrangement was proposed. The inclined load with the given angle \( \alpha \) was applied to the free flange, see Figure 5. Both gravity and uplift loading were simulated in this way. The tests were executed on the same specimens as the standard tests.

In the searched ratio \( k_q = C_D/C_{D,0} \), \( C_D \) represents the stiffness due to the lateral load component from test comprising the load effect and \( C_{D,0} \) represents the stiffness from standard test.

![Figure 5. Tests covering the effect of external purlin load – scheme, test to simulate gravity load.](image)

For the RT1 (screwed down roof) the gravity loading was simulated in 7 tests and the uplift loading in 11 tests. In the 5 of 7 gravity load tests, the reached flexibility was smaller than the flexibility due to purlin deformation \( 1/K_B \). In such a case the \( C_D \) cannot be determined, because the flexibility \( 1/C_D \) is negative. This corresponds well with the theoretical analysis of Vraný (2005). In the remaining 2 tests the gravity load showed the increase of the \( C_D \) (\( k_q = 1.41 \) and 5.38 respectively). Despite the general expectation with the tests representing the uplift loading, 8 tests of 11 showed the \( k_q > 1 \).
The analytical method (Vraný (2005)) determining the effect of the load on $C_D$ for purlins, which is based on the moment equilibrium conditions in the purlin-sheeting area, can be applied also for the spacer systems RT3 and RT4, where one has to determine the values of $k_q$ separately for the rail-sheeting connection (the rotational stiffness at this level is denominated as $C(D(A))$) and for the spacer-purlin connection (the rotational stiffness at this level is denominated as $C(D(C))$).

![Diagram](image)

**Figure 6.** Model of determination of the effect of external load for RT4 (single skin spacer system roof).

In case of the RT4 (single skin roof system with the bridge spacers and rails), the moment at the point of the compression contact of the rail-sheeting connection for uplift load is (see Figure 6):

$$M = F_H (h + h_c) - F_V (b_b + b_{c,A})$$  \hspace{1cm} (9)

Ratio of the moment $M$ to the one due to the lateral load only:

$$\frac{1}{k_{q(A)}} = \frac{F_H (h + h_c) - F_V (b_b + b_{c,A})}{F_H (h + h_c)} = \frac{\tan \alpha (h + h_c) - (b_b + b_{c,A})}{\tan \alpha (h + h_c)}$$  \hspace{1cm} (10)

The moment to the compression contact of the bridge spacer with the purlin for uplift load:

$$M = F_H h - F_V (b_b + b_{c,C})$$  \hspace{1cm} (11)

Ratio of this moment to the one due to lateral load only (relevant for the rotational stiffness $C(D(C))$):

$$\frac{1}{k_{q(C)}} = \frac{F_H h - F_V (b_b + b_{c,C})}{F_H h} = \frac{\tan \alpha h - (b_b + b_{c,C})}{\tan \alpha h}$$  \hspace{1cm} (12)

In case of the RT4a (single skin spacer system with corrugated sheeting, see Figure 1c), 4 tests were done simulating the gravity loading and 3 tests simulating the uplift loading. The gravity load showed smaller flexibility than the flexibility due to the purlin deformation in all tests (similar behaviour as observed in the most RT1 gravity tests).

The uplift loading led to the increase of the rotational stiffness. The test results and comparison with the calculation are shown in Table 1. Compared values are:

- the flexibility $\delta_{exp} = 1/K_{exp}$ from the test with the uplift load,
- the flexibility $\delta_{th}$ calculated from the standard test using the a.m. theoretical procedure:

$$\delta_{th} = \frac{1}{K_B} + \frac{(h + h_c) h_A}{k_{q(A)} C(D(A),0)} + \frac{h (h_A - h_c)}{k_{q(C)} C(D(C),0)}$$  \hspace{1cm} (13)

Last column shows the good agreement of the proposed theoretical procedure with the test results. It should be noted that the result is very sensitive on the chosen rotation points.

**Table 1.** Tests to simulate uplift load for RT4a roof system

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Experiments</th>
<th>Rot. restraints</th>
<th>Factors</th>
<th>Deflections</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>h</td>
<td>$K_{exp,0}$</td>
<td>$K_{exp}$</td>
<td>$K_B$</td>
<td>$C(D(A))$</td>
</tr>
<tr>
<td>31-1</td>
<td>1.70</td>
<td>252.3</td>
<td>2.67</td>
<td>8.29</td>
<td>20.94</td>
</tr>
<tr>
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<td>3.42</td>
<td>8.41</td>
<td>20.36</td>
</tr>
<tr>
<td>31-3</td>
<td>1.70</td>
<td>252.3</td>
<td>3.16</td>
<td>8.58</td>
<td>20.28</td>
</tr>
</tbody>
</table>
4 LATERAL RESTRAINT

4.1 General methodology of analysis

Lateral restraint of the purlin upper flange is caused by the shear stiffness of the roof diaphragm. The shear stiffness assures the mutual restraint of the two points along the purlin length in lateral direction. The analysis is based on the ECCS Recommendations (1995) and the shear flexibility of the fasteners was determined by tests. There is an important effect of the roof slope length (roof dimension perpendicular to the purlin span), representing the depth of the shear diaphragm, on the shear stiffness. For the Astron typical roofs this dimension was limited to minimum 12m. Dimension of the shear diaphragm can be critical for walls.

4.2 Analysis of the spacer roof systems RT3 and RT4

For the spacer roof systems RT3 and RT4 the lateral flexibility of the spacers (omega spacer for RT3 and bridge spacer for RT4) and the thermal expansion clips (equal for both RT3 and RT4) has to be determined. The proposed method is described here below.

![Diagram](image)

Figure 7. Model for the lateral stiffness of the spacers and the thermal expansion clips:

a) RT3a and RT4a spacer roof systems, b) RT3b and RT4b spacer roof systems

The flexibility due to the spacer rotation $c_3$ is defined as the lateral movement of the rail with respect to the top flange of the purlin due to unit lateral force. It can be derived from the rotational restraint tests.

The spacers and the thermal expansion clips are considered as a stiff bodies fixed at the both ends. These fixings are not infinite and the stiffness at the connection to the sheeting is denominated as $C_{D(A)}$, and at the connection to the purlin as $C_{D(C)}$. The distribution of moments is linear.

For the roof systems RT3a and RT4a with the screwed down sheeting, the moment pattern corresponds to the ratio of the end values of the stiffness. The following formulas apply:

$$\phi = \frac{M_A}{C_{D(A)}} = \frac{M_C}{C_{D(C)}}$$  \hspace{1cm} (14a)
\[ M = M_A + M_C = F h_c \]  
(14b)

\[ K_3 = \frac{1}{c_3} = \frac{F}{\delta} = \frac{M}{h_c} \frac{1}{h_c \phi} \phi h_c^2 \left( 1 + \frac{C_{D(C)}}{C_{D(A)}} \right) = \frac{C_{D(A)} + C_{D(C)}}{h_c^2} \]  
(15)

The same attitude is used for the roof systems RT3b and RT4b (see Figure 1d)) with the standing seam sheeting having the thermal expansion clips. The rotations are:

\[ \varphi_A = \frac{M_A}{C_{D(A)}}, \quad \varphi_B = \frac{M_B}{C_{D(B)}}, \quad \varphi_C = \frac{M_C}{C_{D(C)}}, \quad \varphi_A = \varphi_C + \varphi_B \]  
(16)

For the moments we can write:

\[ M = M_A + M_C = F \left( h_c + h_{clip} \right) \]  
(17)

\[ M_B = M_C - \frac{h_c}{h_c + h_{clip}} (M_A + M_C) \]  
(18)

The moments can be evaluated by merging the equations (16), (17) and (18). Set of two equations to evaluate the \( M_A \) and \( M_C \) is obtained and consequently the lateral stiffness \( K_3 \) is determined as:

\[ K_3 = \frac{F}{\delta} = \frac{F}{h_{clip} \cdot \varphi_A + h_c \cdot \varphi_C} \]  
(19)

5 CONCLUSIONS

The basic principles for the determination of the sheeting parameters needed for the design of the Astron cold formed purlins are presented. In addition to the brief outline of the extensive experimental program which is still ongoing in 2009, some specific theoretical approaches were emphasized, namely:

- determination of the test procedures for the non-standard cases of the rotation stiffness,
- principle of the grouping for the tests with the same sheeting but different purlins,
- application of the components method for the determination of the rotational restraint of the spacer roof systems,
- general method for analysis of the effect of the purlin load on the rotational stiffness,
- method for the determination of the lateral stiffness of spacers and thermal expansion clips used with the spacer roof systems.

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