Effect of post yielding stiffness of braces on soft storey mechanism for two storey braced frames with restraint braces

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ABSTRACT: In general, steel concentrically braced frames are designed to resist lateral force by means of truss action. Design considerations for columns in these frames are therefore governed by the column axial force and column moment demands are generally ignored. However, if the columns cannot carry moment, then dynamic inelastic time-history analyses show that a soft-storey mechanism is likely to occur causing large concentrated deformations in only one storey. Such large concentrations of damage are not generally seen in real frames since columns are generally continuous and they possess some flexural stiffness and strength. This paper develops relationships for column property and drift concentration within a frame based on pushover and dynamic analyses. It is shown that continuous seismic and gravity columns in a structure significantly decrease the possibility of large drift concentrations.

1 INTRODUCTION

For braced frames as a shear resistant model, a soft-storey mechanism in the braced frames may occur, but a soft-storey mechanism can be restrained by the continuous columns. The effect of column stiffness on the storey drift concentration of braced frames has been evaluated by MacRae, Kimura and Roeder (2004), and then the material property of braces is assumed to be perfectly elasto-plastic. In the paper, it was shown that if the column flexural stiffness increases, the storey drift concentration decreases. Then, the post yielding stiffness is assumed to be equal to 0 in the analytical model, but the braces in the real frames have strain hardening. The slope of strain hardening depends on the steel grade. The post yielding stiffness of the braces on the first and second storeys in the two storey braced frames may be different due to the steel grade or the influence of initial imperfection of braces. If strength and stiffness on the first and second storeys are different, the seismic behavior of two storey braced frames may be different from that with the same strength and the same stiffness on both storeys. This paper describes the effect of continuous columns of two storey frames with buckling restrained braces having variable strength and stiffness on the first and second storeys. Then the main seismic resistant elements are the braces with the specified post yielding stiffness, and the sub seismic resistant elements are the continuous columns. The effect of the post yielding stiffness on the storey drift concentration is investigated. Equations for estimating the column moment demand and drift concentration factor are developed, and recommendations for column stiffness and strength capacities for a specified drift concentration are then proposed. Finally, dynamic analyses are performed, and equations for the column moment demand and the drift concentration factor are clarified.
2 STATIC BEHAVIOR FOR TWO STOREY BRACED FRAME WITH LEANING COLUMNS

2.1 Development of equations of storey drift and column moment of two storey braced columns

Continuous columns over the height of the frame, whether they are gravity or seismic columns, resist the tendency for concentration of deformation in one storey. The amount of drift concentration will depend on the column flexural stiffness.

The two storey braced frame with an elastic column at pinned base will deform as shown in Figure 1. The lateral force distribution is given from $A_i$ distribution, in Japanese seismic design code (Building Center of Japan 2001), and then the lateral force at Level 2 is about 1.8 times as that at Level 1. After yielding of the braces with strain hardening, most of additional shear forces are carried out by the columns at pinned bases and the other are carried out by the yielded braces. The storey shear forces, $V_{si}$, at Levels $i$, are shown in Eq. (1). The shear force at level 2 is equal to about 0.644 of that at Level 1 ($\varphi = 0.644$).

$$V_{s1} = V_{f1} + V_c, \quad V_{s2} = \varphi V_{s1}, \quad V_{s2} = V_{f2} - V_c$$

$$V_c = 3K_i\alpha \delta_{c-i}$$

$$\alpha = \sum_{i=1}^{n} \frac{E_i l / h^3}{\sum_{i=1}^{n} E_i A_i \cos^2 \theta / l}$$

where $V_{fi}$ show the shear forces for braces, $V_c$ shows the column shear forces, and $\alpha$ is the ratio of the column flexural stiffness to brace axial stiffness. The drift concentration factor, $\gamma$ is developed being classified into three stages such as following.

i) frame displacements before yielding of brace on 1st storey

Before brace yielding, the displacements, $\delta_{1-1}$ and $\Delta_{2-1}$ on each storey are:

$$\delta_{1-1} = \frac{l^3}{E A b^2} V_{f1} \quad \Delta_{2-1} = \delta_{1-1} + \delta_{2-1} = \frac{l^3}{E A b^2} (V_{f1} + V_{f2} + V_{f1y})$$

where $\beta$ is the lateral stiffness ratio over the braced frame height, $E A$ is the axial stiffness of the braces. $l$, $b$, $h$ and $H$ are shown in Figure 1. When braces yield, the total displacement at each level $\delta_{1-1}$ and $\Delta_{2-1}$ is developed from frame shear forces at yield, $V_{f1y}$ and Eqs. (2) ~ (4).

$$\delta_{1-1} = \frac{l^3}{E A b^2} V_{f1y} \quad \Delta_{2-1} = \delta_{1-1} + \frac{l^3}{E A b^2} \frac{2\varphi + 3\alpha(\varphi + 1)}{2\beta + 3\alpha(\varphi + 1)} V_{f1y}$$

The column and the brace shear forces on first storey, $V_{c1}$ and $V_{f2}$ are obtained using yield shear force, $V_{f1y}$ as shown in Eq. (6).

$$\Delta = \mu \Delta_y \quad (\Delta_y: \text{yield displacement})$$

Figure 1. Model of two storey braced frame: (a) Loading and deformation conditions, (b) Relationship between axial force and axial displacement
\[ V_{f1} = \frac{3\alpha(\beta - \varphi)}{2\beta + 3\alpha(\varphi + 1)} V_{f1y}, \quad V_{f2} = \frac{2\varphi + 3\alpha(\varphi + 1)}{2 + 3\alpha(\varphi + 1)/\beta} V_{f1y} \]  

ii) incremental displacements after yielding of braces on first storey to both storey braces yielding

After first storey braces yielding, the shear forces are represented as incremental form in the following.

\[ \Delta V_{s1-2} = \Delta V_{f1-2} + \Delta V_{c2}, \quad \Delta V_{s2-2} = \varphi \Delta V_{s1-2}, \quad \Delta V_{s1-2} = \Delta V_{f2-2} + \Delta V_{c2} \]  

\[ \Delta V_{c2} = 3K_c \alpha \delta_{c2} \]  

\( \delta_{c2} \) and \( \Delta_{c2} \) are obtained using the increment of brace shear forces on each storey, \( \Delta V_{f1-2}, \Delta V_{f2-2} \) of Eqs. (7) and (8) as shown in Eq. (9).

\[ \delta_{c2} = \frac{l^3}{\chi_i EAb^2} \Delta V_{f1-2}, \quad \Delta_{c2} = \delta_{c2} = \frac{l^3}{EAb^2} \left( \frac{\Delta V_{f1-2}}{\chi_i} + \frac{\Delta V_{f2-2}}{\beta} \right) \]  

where \( \chi_i \) is the ratio of post yielding stiffness to elastic stiffness. \( \Delta V_{f1-2}, \Delta V_{f2-2} \) are obtained from Eqs. (7), (8) and (9) using yield shear force, \( V_{f1y} \).

\[ \Delta V_{f1-2} = \chi_i \left( \mu - 1 \right) \left[ 2\beta + 3\alpha(\varphi + 1) \right] \left[ \frac{2\varphi + 3\alpha(\varphi + 1)}{2\beta + 3\alpha(\varphi + 1)} + 1 \right] V_{f1y} \]  

\[ \Delta V_{f2-2} = \left( \mu - 1 \right) \beta \left[ 2\varphi \chi_i + 3\alpha(\varphi + 1) \right] \left[ \frac{2\varphi + 3\alpha(\varphi + 1)}{2\beta + 3\alpha(\varphi + 1)} + 1 \right] V_{f1y} \]  

The increment of lateral displacements on each storey, \( \delta_{c2} \), and \( \Delta_{c2} \) are obtained from Eqs. (9) ~ (11) using \( V_{f1y} \).

\[ \delta_{c2} = \frac{(\mu - 1)\left[ 2\beta + 3\alpha(\varphi + 1) \right]}{2\beta + 3\alpha(\varphi + 1)} \left[ \frac{2\varphi + 3\alpha(\varphi + 1)}{2\beta + 3\alpha(\varphi + 1)} + 1 \right] \frac{l^3}{EAb^2} V_{f1y} \]  

\[ \Delta_{c2} = (\mu - 1) \Delta_{c2} = (\mu - 1) \left[ \frac{2\varphi + 3\alpha(\varphi + 1)}{2\beta + 3\alpha(\varphi + 1)} + 1 \right] \frac{l^3}{EAb^2} V_{f1y} \]  

And the increment of column shear force on each storey, \( \Delta V_{c2} \) is given using \( V_{f1y} \) as shown in Eq. (14).

\[ \Delta V_{c2} = \frac{3\alpha(\mu - 1)(\beta - \varphi \chi_i)}{2\beta + 3\alpha(\varphi + 1)} \left[ \frac{2\varphi + 3\alpha(\varphi + 1)}{2\beta + 3\alpha(\varphi + 1)} + 1 \right] V_{f1y} \]  

iii) incremental displacements after both storeys braces yielding

After both storeys braces yielding, the shear forces are represented as incremental form in the following such as case ii).

\[ \Delta V_{s1-3} = \Delta V_{f1-3} + \Delta V_{c3}, \Delta V_{s2-3} = \varphi \Delta V_{s1-3}, \Delta V_{s2-3} = \Delta V_{f2-3} - \Delta V_{c3} \]  

\[ \Delta V_{c3} = 3K_c \alpha \delta_{c3} \]  

When both storey braces yield, relationship between the shear forces and the yield shear force on second storey, \( V_{f2y}, \Delta V_{f2-2} \) and \( V_{c2} \) are given in the following.

\[ V_{f2y} - V_{f2} = \Delta V_{f2-2} \]  

The ductility, \( \mu_s \) at both storeys yielding is obtained from Eqs. (7), (8), (11) and (17).

\[ \mu_s = 1 + \frac{2\varphi \chi_i + 3\alpha(\varphi + 1)}{2\varphi + 3\alpha(\varphi + 1)} \left[ \varphi + \beta + 3\alpha(\varphi + 1) \right] \]  

In \( \mu_s < \mu \), the incremental lateral displacements on each storey, \( \delta_{c1}, \Delta_{c1} \) are given using the incremental shear forces on each storey, \( \Delta V_{f1-3}, \Delta V_{f2-3} \).
\[ \delta_{1-3} = \frac{I^3}{E_i A E b^2} \Delta V_{f1-3}, \Delta_{2-3} = \delta_{1-3} + \delta_{2-3} = \frac{I^3}{E_i A b^2} \left( \frac{\Delta V_{f1-3}}{X_i} + \frac{\Delta V_{f2-3}}{X_i \beta} \right) \]  

(19)

\[ \Delta V_{f1-3}, \Delta V_{f2-3} \] are obtained from Eqs. (15), (16) and (19) using yield shear force, \( V_{f_{1y}} \) in the following.

\[ \Delta V_{f1-3} = \frac{(\mu - \mu_i) X_i}{2(\beta X_1 + \phi X_1 + 3a(\phi + 1))} \left\{ \frac{2\phi + 3a(\phi + 1)}{2\beta + 3a(\phi + 1)} +1 \right\} V_{f_{1y}} \]  

(20)

\[ \Delta V_{f2-3} = \frac{\beta X_1 (\mu - \mu_i)}{2(\beta X_2 + \phi X_1 + 3a(\phi + 1))} \left\{ \frac{2\phi + 3a(\phi + 1)}{2\beta + 3a(\phi + 1)} +1 \right\} V_{f_{1y}} \]  

(21)

The increment of lateral displacements on each storey, \( \delta_{1-3} \), and \( \Delta_{2-3} \) are obtained from Eqs. (19) ~ (21) using \( V_{f_{1y}} \).

\[ \delta_{1-3} = \frac{(\mu - \mu_i) X_i}{2(\beta X_2 + \phi X_1 + 3a(\phi + 1))} \left\{ \frac{2\phi + 3a(\phi + 1)}{2\beta + 3a(\phi + 1)} +1 \right\} \frac{I^3}{E_i A b^2} V_{f_{1y}} \]  

(22)

\[ \Delta_{2-3} = \left( \mu - \mu_i \right) \Delta_{2-3} = \left( \mu - \mu_i \right) \left\{ \frac{2\phi + 3a(\phi + 1)}{2\beta + 3a(\phi + 1)} +1 \right\} \frac{I^3}{E_i A b^2} V_{f_{1y}} \]  

(23)

And the increment of column shear force on each storey, \( \Delta V_{c-3} \) is given using \( V_{f_{1y}} \) as shown in Eq. (24).

\[ \Delta V_{c-3} = \frac{3a(\mu - 1)(\beta X_2 - \phi X_1)}{2(\beta X_2 + \phi X_1 + 3a(\phi + 1))} \left\{ \frac{2\phi + 3a(\phi + 1)}{2\beta + 3a(\phi + 1)} +1 \right\} V_{f_{1y}} \]  

(24)

iv) development of drift concentration factor and column moment

The total lateral displacement on each storey is given as shown in Eq. (25).

\[ \delta_i = \sum_{i=1}^{N} \delta_{i-1}, \Delta_2 = \sum_{i=1}^{N} \Delta_{2-i} \]  

(25)

where \( N \) shows the stage of frame deformation such as i) ~ iii). The drift concentration factor, \( \gamma \) is given from Eq. (26).

\[ \gamma = \frac{\text{MAX} \left( |\Delta_2 - \delta_i|, \delta_i \right)}{\Delta_2 / h} \]  

(26)

The column moment at each stage is obtained from the Eqs. (6), (14) and (24) for shear forces.

\[ M_i = \sum_{i=1}^{N} V_{c-3} h \]  

(27)

2.2 Drift concentration for two storey brace frames

Figure 2 (a) and (b) compare actual and theoretical relationships for drift concentration factor, \( \gamma \) for specified post yielding stiffness of first storey braces, \( X_1 \). The frame shear strength ratio, \( \beta \), is 5/6, and post yielding stiffness of second storey braces, \( X_2 \), is 0.01. Analysis A shows \( \gamma \) from Eq. (26).

These curves shown depend on which equation governs. Analysis B shows \( \gamma \) from a frame analysis using the computer program DRAIN-2DX (Prakash et al. 1993). Here, the beam and column members were made rigid axially, and the frame members were pinned at their ends. Analyses A and B are consistent. In Figure 2 (a) and (b), the kink in the curve shows where braces behavior changes from first storey only to both storeys yields. Its value is different among 4 kinds of curves. The value of frame ductility, \( \mu \) at the kink does not depend on the magnitude of \( X_1 \), and 4 kinds of curves almost same for 0.1 of \( \alpha \) in Figure 2 (b), even though they are different for 0.01 of \( \alpha \) in Figure 2 (a). In Figure 2 (a) and (b), the soft storey mechanism occurs when only first storey braces yield. The drift on second storey increases after second storey braces yield, and then \( \gamma \) suddenly reduces. As \( X_1 \) is greater, \( \gamma \) reduces. For \( X_1 \) of 0.1 and \( \mu \) over about 5, the drift concentrates on the second storey, and \( \gamma \) increases.
Figure 3 (a) and (b) compare actual and theoretical relationships for drift concentration factor, $\gamma$ for specified frame ductility, $\mu$. The frame shear strength ratio, $\beta$, is 5/6, and the post yielding stiffness of second storey braces, $\chi_2$, is 0.01. For $\mu$ of 1 which means just the first storey braces yielding, the drift concentration factor does not depend on the post yielding stiffness of first storey braces, and that is constant. In Figure 3 (a), second storey braces remain elastic till $\mu$ of 2. From Figure 2 (a) and (b), when only the first storey braces are yielding, the value of $\gamma$ is mostly same not to depend on the post yielding stiffness of the first storey braces. So the post yielding stiffness of the first storey braces, $\chi_1$, has a very small effect for drift concentration factor. For $\mu$ of 4 and $\chi_1$ over about 0.02, the second storey braces yield. And the drift on the second storey increases, $\gamma$ suddenly reduces. For $\mu$ of 6 and $\chi_1$ over about 0.08, the drift concentrates on the second storey, and $\gamma$ increases.

Figure 4 (a), compares actual and theoretical relationships for drift concentration factor, $\gamma$ for the specified post yielding stiffness of first storey braces, $\chi_1$. Figure 4 (b), compares actual and theoretical relationships for drift concentration factor, $\gamma$ for the specified post yielding stiffness of the second storey braces, $\chi_2$. The frame shear strength ratio, $\beta$, is 5/6, and frame ductility, $\mu$ is 4. As would be expected, $\gamma$ tends to unity as the column flexural stiffness increases. It may be seen from Figure 4 (a) that the column is effective in reducing the storey drift concentration when the column stiffness ratio, $\alpha$, is greater than about 0.1. As the magnitude of the post yielding stiffness of the first storey braces, $\chi_1$, is greater, $\gamma$ tends to reduce in $\alpha$ of less than 0.1. In Figure 4 (b), when the value of $\alpha$ is smaller than about 0.01, the second storey braces remain elastic, so that the value of $\gamma$ is same not to depend on the post yielding stiffness of the second storey braces. When $\alpha$ is larger than about 0.01, both of storey braces yield. For $\alpha$ from 0.01 to 0.1, as $\chi_2$ is larger, the drift on the second storey becomes small. On the other hand, the drift on the first storey becomes large, so that $\gamma$ increases.
2.3 Column moment for two storey brace frames

Figure 5 (a) and (b) compare actual and theoretical relationships for column maximum moment demand for the specified first storey braces post yielding stiffness, $\chi_1$ from Eq. (27). The frame shear strength ratio, $\beta$, is 5/6, and the post yielding stiffness of second storey braces, $\chi_2$, is 0.01. In Figure 5 (a) and (b), the kink in the curve shows where braces behavior changes from the first storey only to both storeys yields. The value of column maximum moment demand does not depend on value of $\chi_1$ when only first storey braces yields. As $\chi_1$ is greater, the magnitude of ductility demand at the second braces yielding is smaller and the slope after the kink becomes higher. The slope after the kink becomes plus when $\chi_1$ is lower than $\chi_2$, and the slope after the kink becomes minus when $\chi_1$ is larger than $\chi_2$. For 4 kinds of $\chi_1$, the values of the moment demand at both storey braces yielding are different in Figure 5 (a), but they are almost same in Figure 5 (b).

Figure 6 (a) and (b) compares actual and theoretical relationships for column maximum moment for the specified frame ductility, $\mu$. The frame shear strength ratio, $\beta$, is 5/6, and the post yielding stiffness of second storey braces, $\chi_2$, is 0.01. In Figure 6 (a) and (b), column maximum moment demand is almost constant when only first storey braces yield. After both of storeys braces yield, the column maximum moment demand for the high post yielding stiffness of first storey braces is low.

Figure 7 (a) compares actual and theoretical relationships for column maximum moment for specified post yielding stiffness, $\chi_1$ of both storey braces. Figure 7 (b) compares actual and theoretical relationships for column maximum moment demand for the specified post yielding stiffness of the first storey braces, $\chi_1$. Figure 7 (c) compares actual and theoretical relationships for column maximum moment demand for the specified post yielding stiffness of the second storey braces, $\chi_2$. In Figure 7 (a) when values of the column stiffness ratio, $\alpha$ are less than about 0.1, the column moment demand for the small post yielding stiffness is higher than that for the large stiffness. And when values of $\alpha$ are greater than about 0.1, the column moment demand for the low post yielding stiffness is smaller than that for the high stiffness. In Figure 7 (b), the column maximum moment demand for the high post yielding stiffness of the first storey braces is lower than that for the small stiffness. In Figure 7 (c), when the value of $\alpha$ is smaller than 0.01, the second storey braces remain elastic. And column maximum moment is identical not to depend on the value of $\chi_2$.

![Figure 5](image1.png)
![Figure 6](image2.png)
![Figure 7](image3.png)
of $\chi^2$. When the value of $\alpha$ is larger than 0.01, the second storey braces are yielding. So that the column maximum moment for the high post yielding stiffness of the first storey braces is larger than that for the lower stiffness. Columns should be designed for this maximum moment according to Eq. (27).

3 DYNAMIC BEHAVIOR FOR TWO STOREY BRACED FRAMES WITH LEANING COLUMNS

In the previous section, the equations for the static drift concentration factor were developed for the various brace post yielding stiffness and the column flexural stiffness assuming the lateral force distribution for Japanese seismic design code. Two storey braced frame with a 3600 mm interstorey height, $h$, and a bay width, $B$ of 9000mm is to designed such as Figure 1. During an earthquake the inertial forces are constantly changing, so that the drift concentration factor, $\gamma$ may be different from that described above. These frames with $\beta=1, 5/6, 2/3$, and with $\chi_i=0.001, 0.01, 0.05, 0.1$ were selected and the earthquake records were used with El Centro NS/EW Hachinohe NS/EW Kobe NS/EW Taft NS/EW and Tohoku NS/EW whose maximum value of velocity was 50 kine and 75 kine, in order to investigate the effect of the column stiffness under large lateral deformation of the braced frames after braces yielding.

Damping was applied as 2% of critical in the first and second mode using a Rayleigh damping model. The kinematic hardening law is applied to hysteresis loop of braces. Newmark’s constant average acceleration integration method was used in the inelastic dynamic time history analysis. The first mode response period for the two storey frame was about 0.332s.

Figure 8 (a)–(c) show the time history of the ductility on the first storey whose maximum storey drift occurs. For $\alpha=0.01$ and $\chi=0.001$, the value of maximum ductility arrives at 4 and the residual deformation occurs. The value of maximum ductility for $\alpha=1$ and $\chi_i=0.001$, is lower than that for $\alpha=0.01$ and $\chi_i=0.001$ because of high column stiffness. On the other hand, the residual deformation also occurs. For $\alpha=0.01$ and $\chi_i=0.1$, the maximum ductility and residual displacement are smallest in 3 kinds of frames.

Figure 9 (a)–(c) show the time history of column moment demand on the first storey whose maximum column moment occurs. The maximum column moment demand for $\alpha=1$ and $\chi_i=0.001$ is larger in 3 kinds of frames. It is shown that the column stiffness is high, its magnitude is larger.

Figure 10 compares the static and dynamic drift concentration factor, $\gamma$ and $\gamma_d$. It may be seen that the $\gamma_d$ mostly consists with the $\gamma$, so that the $\gamma_d$ can also be estimated using Eq. (26).

Figure 11 compares the static and dynamic column moment, $M_c$ and $M_{cd}$ whose value is defined as that when the maximum drift occurs. It may be seen that the $M_{cd}$ mostly consists with the $M_c$, so that the $M_{cd}$ can also be estimated using Eq. (27) except 0.014, 0.11 and 0.22 of $M_c/V_f1y_h$. The reason why the dynamic and static column moment, $M_{cd}$ and $M_c$ are different at 0.014, 0.11 and 0.22 of $M_c/V_f1y_h$ against moment is the following. In dynamic analysis, the column moment that increases or decreases is due to inertia force. On the other hand, in static analyses, the moment becomes almost constant, even though the column stiffness ratio, $\alpha$ changes such as Figure 7.
CONCLUSIONS

Closed-form equations were derived to estimate the drifts for the applied forces based on equilibrium, compatibility and constitutive relations. Pushover and dynamic analyses were then performed to investigate the soft storey mechanism of the two storey braced frames with elastic columns. It was shown that:

1) The deformation of two storey braced frames consists of three stages which are i) elastic frame response before yield, ii) only one storey yielding, and iii) both storeys yielding. The drift concentration factor and the column moment demand for the three stages above can be found from Eqs. (26) and (27).

2) The magnitude of the drift concentration factor or the column moment demand is influenced by the column stiffness ratio and the post yielding stiffness of braces.

3) The peak drift concentration factor during shaking and dynamic column moment demand can be estimated as the static drift concentration factor and static column moment demand.

REFERENCES