Influence of ballast superstructure on the dynamics of slender steel railway bridges

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ABSTRACT: In this paper, the influence of the ballast superstructure on the vibration characteristics of single-span beam bridges is investigated. In order to analyse the mechanisms and the interaction dynamics between the ballast superstructure and the bridge structure, an experimental bridge with a span width of 10m was built carrying the standard assembly of ballast superstructure used by the Austrian federal railway company. The dynamic properties of the bridge were tested thoroughly using a set of two eccentric weight vibration generators. Basing on these experimental results, a mathematical (mechanical) model is developed which describes the effect of the ballast superstructure on the dynamic behaviour of the bridge. This model is formulated in a way that it can be adopted for other railway bridges as well. An example is given for a real life bridge where the dynamic characteristics are analysed with and without the influence of the ballast superstructure using the proposed mathematical model.

1 INTRODUCTION

In the dynamic analysis of railway bridges discrepancies between calculation results and test results are frequently observed (cf. Zabel & Brehm (2008)). In many cases, the dynamic response of bridges under moving loads is overestimated and the predictions made are too conservative (cf. Winselmann & Fahlbusch (2006)). This problem especially applies to relatively short railway bridges (span width below 30m) with ballast superstructure. So far, there are no means provided in the current design codes which allow taking the ballast superstructure into account when a dynamic analysis has to be conducted.

In order to investigate the influence of ballast superstructure on the dynamic properties of railway bridges, the Institute for Structural Engineering at the Technical University of Vienna developed a steel bridge for testing purposes. Modal tests have been carried out with a system of two vibration generators to identify the eigenfrequencies and the damping (cf. Mähr & Fink (2008)). The results of the modal tests show that the ballast superstructure provides additional damping and stiffness to the bridge and have been published previously (cf. Fink & Mähr (2008), Mähr (2009)). In the current paper, a mathematical model is presented which describes the effect of the ballast superstructure on the dynamic behaviour of the experimental bridge. This model can be adopted for other railway bridges as well. This is shown in an example for a real life bridge, where the dynamic response under moving loads is analysed using the proposed model. The example shows that the ballast superstructure has a strong influence on the damping but only a weak influence on the stiffness of the whole bridge.
2 MODAL TESTS: STEEL BRIDGE WITH BALLAST SUPERSTRUCTURE

2.1 Preface

The bridge was designed for testing purposes only and is set up as a straight single-span beam bridge spreading 10.0m, s. Figure 1 on the left.

There are two longitudinal main girders consisting of European wide flange beams HEA 340. Crossbeams are welded bending resistant to the main beams to ensure adequate horizontal stiffness of the bridge. The bridge deck is made of timber beams and panels, whereupon the standard assembly of ballast superstructure used by the Austrian federal railway company is built. It consists of a gravel bed with a thickness of 55cm, reinforced concrete sleepers and rails type UIC 60. The bridge is supported by reinforced concrete bases rising approx. 50cm. Four elastomer pads are used as bearings between the main girders and the bases. The modal properties of the bridge are analysed by means of a system of two excentric weight vibration generators, s. Figure 1 on the right. The vibration generators induce a harmonically varying force into the bridge and the dynamic response is measured with several capacitive accelerometers and inductive displacement transducers. The harmonic force \( P(t) \) is calculated from the (static) momentum \( m_e \) and the excitation frequency \( \Omega \) [rad/s]:

\[
P(t) = m_e \Omega^2 \sin(\Omega t)
\]

(1)

The momentum \( m_e \) is defined by the product of the total excentric mass \( m_e \) [kg] and the excentricity \( e \) [m]: \( m_e = m_{ex} e \). The momentum of each vibration generator can be changed stepwise ranging from 0 to 15.97kgm. This is done by placing various numbers of weights in the baskets, s. Figure 1 on the left. Frequency-sweeps are carried out to obtain frequency response functions (FRF). The excitation frequency is increased step-by-step and the amplitudes of steady-state vibration of the bridge are measured. The steady-state displacement response \( w(\Omega) \) due to harmonic excitation of unit magnitude at the frequency \( \Omega \) is called a receptance-FRF (abbr.: rec). When the FRF deals with acceleration per unit force, it is called an accelerance FRF (abbr.: acc). For example, the receptance and accelerance in midspan is calculated from the transverse displacement \( w_{zm} \) and acceleration \( \ddot{w}_{zm} \) respectively and the harmonic force excitation at frequency \( \Omega \):

\[
H_{w_{zm}/p}^{rec}(\Omega) = \frac{w_{zm}(\Omega)}{|P(\Omega)|} \quad H_{w_{zm}/p}^{acc}(\Omega) = \frac{\ddot{w}_{zm}(\Omega)}{|P(\Omega)|}
\]

(2)

The simultaneous usage of two vibration generators permits a more accurate identification of the modal properties following the concept of multipoint excitation (also known as ‘normal mode testing’ or ‘appropriation’, Ewins (2000)). A more detailed description of the vibration generators and the way they are used for modal testing is given in Fink & Mähr (2008), Mähr (2009).

The influence of the ballast superstructure on the dynamic properties of the bridge is detected using a simple approach: First, the bridge is tested without the ballast superstructure and a mathematical model is developed to describe the dynamic behaviour. After that, the ballast superstructure is built in and the tests are repeated. Out of the difference between the two test results it is possible to deduce the dynamic effect of the ballast superstructure. It has to be ensured that the way of inducing force into the bridge is the same in both cases (with and without ballast superstructure). This problem has been overcome by welding frameworks made of steel upon the main beams, which ensure a force-fit connection between the vibration generators and the main girders, s. Figure 1 on the left.
2.2 Results for the bridge without ballast superstructure

In this paper, the investigations focus upon the first bending mode of vibration. The tests have been carried out with both vibration generators mounted in midspan. The total mass of the bridge – including the vibration generators – amounts to 10500 kg. Due to the light damping of the bridge, only the smallest momenta could be used to carry out the frequency-sweeps: \( m_{st} = 0.547 \text{ kgm} \) and \( m_{st} = 0.929 \text{ kgm} \) respectively. In Figure 2, the receptance-FRF (on the left) and the accelerance-FRF (on the right) are shown.

![Figure 2. Receptance-FRF and accelerance-FRF in midspan without ballast superstructure for two different momenta \( m_{st} \).](image)

The FRFs, which were obtained with different momenta, are nearly congruent. Thus, the dynamic behaviour of the bridge is almost linear. The eigenfrequency of the first bending mode of vibration is found to be 4.66 Hz. The damping factor \( \zeta \) (percentage of critical damping) is estimated using the half-power bandwidth method and amounts to 0.85% on average.

2.3 Results for the bridge with ballast superstructure

After the ballast superstructure has been built in, frequency-sweeps could be carried out using much larger momenta: six different momenta were chosen starting from 1.91 kgm up to 7.61 kgm. The results are shown in Figure 3.

![Figure 3. Receptance-FRF and accelerance-FRF in midspan with ballast superstructure for different momenta \( m_{st} \).](image)
The shapes of the FRFs, which were obtained with different momenta, suggest characteristic properties of the bridge. The main lobe of the FRFs shifts towards lower frequencies when the momentum is increased. Resonance appears in the range of 3.84Hz and 4.32Hz. The lobes of the FRFs are inclined towards lower frequencies. This phenomenon indicates nonlinear behaviour of the system. The damping effect of the ballast superstructure is reflected in lower peak values and broadened FRF-lobes, compared to the results of the bridge without ballast superstructure.

In the following chapter, a mathematical model is presented which describes the effect of the ballast superstructure on the bridge.

3 MATHEMATICAL MODEL FOR THE BALLAST SUPERSTRUCTURE

3.1 Model for the bridge without ballast superstructure

The bridge structure without ballast superstructure is modelled using a simply supported Bernoulli-Euler beam with constant properties along the span width, s. Figure 4.

![Beam model for the bridge without ballast superstructure.](image)

Table 1. Data for the beam model in Figure 5.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>10.0</td>
<td>[m]</td>
<td>span width</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>635</td>
<td>[kg/m]</td>
<td>mass per unit length of the bridge structure</td>
</tr>
<tr>
<td>$\mu_{II}$</td>
<td>3099</td>
<td>[kg/m]</td>
<td>mass per unit length of the ballast superstructure</td>
</tr>
<tr>
<td>$\zeta_l$</td>
<td>0.85%</td>
<td>[]</td>
<td>damping factor of the bridge without ballast superstructure</td>
</tr>
<tr>
<td>$\omega_{2z,l}$</td>
<td>1.1631·10^{-10}</td>
<td>[Nm^{-2}]</td>
<td>bending stiffness of the two main girders HEA340</td>
</tr>
<tr>
<td>$M_1$</td>
<td>481</td>
<td>[kg]</td>
<td>mass of the framework in the first an third quarter</td>
</tr>
<tr>
<td>$M_2$</td>
<td>3193</td>
<td>[kg]</td>
<td>mass of the framework and the vibration generators in midspan</td>
</tr>
<tr>
<td>$M_g$</td>
<td>10.5</td>
<td>[t]</td>
<td>total mass of the bridge including the vibration generators, without ballast superstructure</td>
</tr>
<tr>
<td>$M_{gs}$</td>
<td>41.5</td>
<td>[t]</td>
<td>total mass of the bridge including the vibration generators, with ballast superstructure</td>
</tr>
</tbody>
</table>

The partial differential equation of motion (3) is transformed into an ordinary differential equation of motion following the procedure of Ritz and Galerkin. The Ritz approximation for the beam deflection $w_i(x,t)$ satisfying the boundary conditions has the form
The Ritz approximation (4) is substituted into equation (3), then multiplied by \( \phi_i(x) \) and integrated over the path domain of \( w_i(x,t) \), the span width \( L \). The following ordinary differential equation of motion is obtained:

\[
\left( \mu_i \frac{L}{2} + M_1 + M_z \right) \ddot{q}_i + c_{z,i} \frac{L}{2} \dot{q}_i + \left( \frac{EA_{z,i} \pi^4}{2L^4} \right) q_i = P \cos(\Omega t)
\] (5)

The generalised mass amounts to

\[
m_i = \left( \mu_i \frac{L}{2} + M_1 + M_z \right) = 6849 \text{ kg}
\] (6)

and the generalised stiffness amounts to

\[
k_i = \left( \frac{EA_{z,i} \pi^4}{2L^4} \right) = 5.6648 \cdot 10^6 \text{ N/m}
\] (7)

The damping coefficient \( c_{z,i} \) is identified using frequency response analysis technique (Craig & Kurdila (2006)). It is calculated from the peak response obtained in the experiments, s. Figure 2 and is \( c_{z,i} = 606.1 \text{ Ns/m} \).

3.2 Model for the bridge with ballast superstructure

On the basis of the experimental results described in section 2, the ballast superstructure is modelled using a Bernoulli-Euler beam which is connected to the beam of the bridge structure, s. Figure 6.

Between the two beams, a continuous nonlinear visco-elastic bond (shear force-slip relation) is considered. Longitudinal vibrations of the beams are not taken into account. Thus, the shear force \( T \) is found to be

\[
T = k_i \left( w_{i,i} r_i + w_{ii,i} r_{ii} \right) + c_i \left( \dot{w}_{i,i} r_i + \dot{w}_{ii,i} r_{ii} \right) \quad \text{considering} \quad w_{x} = -\varphi
\] (8)

Relative displacements in transverse direction between the two beams are ruled out:

\[
w_j(x,t) \equiv w_{yj}(x,t)
\] (9)

In the following, the variable \( w(x,t) \) is written without index and specifies the displacement of both beams. Therefore, equation (8) may be written

\[
T = k_i \dot{w} \left( r_i + r_{ii} \right) + c_i \ddot{w} \left( r_i + r_{ii} \right)
\] (10)

The equation of motion for the bridge structure follows from equation (3) and has to be completed with the shear force \( T \):

\[
EA_{z,i} w_{xxx} + c_{z,i} \ddot{w} + \mu_i \ddot{w} + M_i \ddot{w}(L/2) \delta(x - L/2) + M_z [\ddot{w}(L/4) \delta(x - L/4) + \ddot{w}(3L/4) \delta(x - 3L/4)] = P \cos(\Omega t) \delta(x - L/2) + T_{x,i} r_i
\] (11)
The bending stiffness $EA_{z,l}$ and the mass per unit length $\mu_z$ are specified in Table 1. The bending stiffness $EA_{z,l}$ of the ballast superstructure (represented by beam $II$) is determined by the bending stiffness of the two rails UIC60. There is no damping coefficient $c_{z,II}$ estimated for beam $II$. The damping effect of the ballast superstructure is merely taken into account via the nonlinear visco-elastic bond between the two beams. The mass per unit length $\mu_\text{II}$ includes the complete ballast superstructure with rails, gravel bed and sleepers. The equation of motion for beam $II$ is

$$EA_{z,II}w_{z,xxx} + \mu_\text{II} \ddot{w} = T_x r_{II}$$

(12)

The equations (11) and (12) are combined to obtain a single equation:

$$EA_{z}w_{z,xxx} - (r_z + r_\mu)^2 (k_z w_{z,xx} + c_z w_{z,x} + c_z \dot{w} + \mu \ddot{w} + M_z \ddot{w} / 2) \delta(x - L / 2)$$

$$+ M_z [\ddot{w} (L / 4) \delta(x - L / 4) + \ddot{w} (3L / 4) \delta(x - 3L / 4)] = P \cos(\Omega t) \delta(x - L / 2)$$

(13)

wherein

$$EA_{z} = EA_{z,l} + EA_{z,II}$$

$$c_z = c_{z,l}$$

$$\mu = \mu_l + \mu_\text{II}$$

(14)

The numerical values for the mechanical properties in equation (14) are listed in Table 2.

### Table 2. Mechanical properties of the beam model for the bridge structure and the ballast superstructure.

| $EA_z$ [Nm$^2$] | Beam I bridge structure | Beam II ballast superstructure | Sum
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1631·10$^8$</td>
<td>1.2831·10$^8$</td>
<td>1.2914·10$^8$</td>
<td></td>
</tr>
<tr>
<td>$c_z$ [Ns/m$^2$]</td>
<td>606.1</td>
<td>-</td>
<td>606.1</td>
</tr>
<tr>
<td>$\mu$ [kg/m]</td>
<td>635</td>
<td>3098.9</td>
<td>3733.9</td>
</tr>
</tbody>
</table>

Equation (13) together with equations (11) and (14) define an equivalent beam model for the whole experimental bridge. The terms describing the visco-elastic bond may be interpreted as continuous rotational spring and damper bedding with coefficients $k_\phi$ and $c_\phi$:

$$k_\phi \equiv k_z (r_z + r_\mu)^2$$

$$c_\phi \equiv c_z (r_z + r_\mu)^2$$

(15)

The nonlinear rotational spring $k_\phi$ and rotational damper $c_\phi$ are identified from the test results. This is accomplished in two steps: First, the rotational spring $k_\phi$ is identified by means of the measured resonance frequencies, see Figure 3. The measured data are fitted best using a quadratic polynomial. Therefore, a quadratic polynomial is also used to determine the rotational stiffness coefficient $k_\phi$:

$$k_\phi = a + b |\phi| + c \phi^2$$

(16)

Substituting equation (16) into the second term of equation (13) and applying the procedure of Ritz and Galerkin as before (using equation (4)) yields a non-linear ordinary differential equation. This equation is solved approximately using a harmonic function (Klotter 1980). The unknown coefficients $a$, $b$, and $c$ are determined by means of curve-fitting using the least square method. The coefficients are fitted in a way that the resonance frequencies obtained from the tests are met. The characteristic curve of $k_\phi$ is plotted in Figure 6 on the left versus the rotation angle $\phi$. It also shows the limits of the rotation angle $\phi_{\text{max}}$ achieved in the tests.

The magnitude of the amplitudes is restricted by the damping capacity, which is assumed to depend on the speed of the rotation angle $\phi$:

$$c_\phi = d + d |\phi|$$

(17)

In contrast to the quadratic polynomial of $k_\phi$, a linear fit of $c_\phi$ is sufficient. At resonance, the coefficients $d$ and $e$ are chosen in a way that the error calculated from the measured deflections and the simulated deflections is as small as possible. The characteristic curve of $c_\phi$ is depicted in Fig. 6 on the right versus the rate of the rotation angle $\dot{\phi}$. It also shows the limits of the rate of the rotation angle $\dot{\phi}_{\text{max}}$. 

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4 EXAMPLE: APPLICATION OF THE MATHEMATICAL MODEL

The mathematical model for the ballast superstructure can be applied to other beam bridges as well. So far, this model is amenable for bridges with a span width of 10m, but further test results with different span widths would allow complementing the model without difficulties. In the following example, the effect of the model on a railway bridge with real life properties shall demonstrate the relevance of the findings when the dynamic response under moving loads has to be calculated.

A trough bridge with a span width of 10m is chosen: total mass (including the mass of the ballast) \( m_B = 11750 \text{ kg} \), bending stiffness \( E A = 9.5540 \times 10^6 \text{ Nm}^2 \). In Eurocode 1, Part 2 (2003), values of damping \( \zeta \) are given for design purposes according to specific bridge systems. A steel bridge with a single span of 10m is assumed to have a damping factor (percentage of critical damping) of 1.75%. This value represents a lower bound estimate and is used no matter which track superstructure is planned. The maximum design speed \( v_{D,\text{max}} \) shall be 250km/h (69.4m/s) and the load model HSLM-A2 according to Eurocode 1 part 2 (2003) is used, which consists of a series of 17 concentrated forces with 200kN. The dynamic response of the bridge under the load model HSLM-A2 is calculated in the time domain for speeds up to 250km/h. The maximum displacement response and the maximum acceleration response are plotted in Figure 7 versus the travelling speed. The dashed lines show the results including the mass of the ballast and using the proposed damping factor of 1.75%.

Figure 6. Characteristic curve of the rotational stiffness (left) and the rotational damping (right).

Figure 7. Displacement spectrum (left) and acceleration spectrum (right) of a 10m trough bridge with and without the effect of the ballast superstructure. Load model HSLM-A2.
The continuous lines show the results including the proposed model for the ballast superstructure, where the additional damping and stiffness effects are considered. The diagrams in Figure 8 reveal the effect of the ballast superstructure on the damping and stiffness properties of the bridge. The damping effect causes a reduction of the maximum values of displacement up to -10.2% and of the maximum values of acceleration up to -18.3%. In the vicinity of resonance, the damping effect of the ballast superstructure is most distinct. The stiffness effect of the ballast superstructure is comparably low and not particularly relevant to real life structures. This is due to the much higher bending stiffness of the trough bridge compared to the bending stiffness of the experimental bridge. In the diagrams of Figure 7, the stiffness effect of the ballast superstructure may be recognised from the shift of the two curves. The additional stiffness increases the resonance frequencies.

To sum it up, the example shows that the ballast superstructure has a significant effect on the dynamic response of short, lightweight railway bridges and should be taken into account when moving load problems are treated.

5 DISCUSSION

The influence of the ballast superstructure on the vibration characteristics of single span beam bridges is investigated. An experimental steel bridge with a span-width of 10m is presented where a full-scale ballast superstructure is built in. The dynamic properties of the bridge without and with the ballast superstructure mounted are gained from modal tests using a system of two vibration generators. Basing on the test results, a mathematical model is proposed to describe the non-linear influence of the ballast superstructure on the bridge. The model consists of two beams: one for the bridge and one for the ballast. Between the beams, a non-linear, visco-elastic shear bond is specified. This model can also be implemented in FE-programs which feature non-linear stiffness and damping elements. The characteristic curves of the stiffness- and damping coefficients are presented. In an example, the relevance of the model is shown when dealing with moving loads. The example demonstrates that the damping effect of the ballast superstructure is significant whereas the stiffness effect can be neglected. Additional tests on bridges with different span-widths should be undertaken to expand the capacity of the proposed model. Also, the effect of continuous ballast superstructure has to be examined as well as higher-order modes of vibration.

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7 REFERENCES