Stress concentration factors for the fatigue design of tubular flange connections

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ABSTRACT: The “hot spot stress” method is currently being used for the design against fatigue by crack propagation from the weld toes of tubular connections. The latter approach requires having a means for obtaining stress concentration factors (SCF) at the critical points (“saddle” and “crown” points), which are usually given by parametric formulae either based on experimental measurements or finite element analyses. To permit applying the method to tubular flange connections, this paper addresses the determination of the SCF at the toe of the tube to flange weld and of the tensile forces induced by fatigue type loading in the bolts. An analytical model, based on elastic theory of thin plates and shells, is developed to calculate the weld toe SCF as well as the induced forces in the bolts. Results obtained via this model and also by an existing French SCF formula are compared to experimental and numerical data for a range of connection geometries and good agreement is observed.

1 INTRODUCTION

Flange joints are used for base supports and continuity joints of tubular members in a variety of structures such as tubular trusses, communication towers made of tubular members, chimneys, pylons for wind turbines and ski-lift installations as well as lighting and road signal poles. While much research, both experimental (Igarashi et al 1985) and theoretical (Kato & Hirose 1985, Igarashi et al 1985, Cao & Packer 1997), has been published on the ultimate resistance of flange joints, less attention has been given to their fatigue resistance although it may be an important design consideration for some if not all the cases mentioned.

The flange joint parts most sensitive to fatigue failure are the tube wall, by cracking from the weld-toe of the tube to flange welded joint, and the bolts which are either connecting two flanges together or connecting a flange to a foundation. Therefore for fatigue design one needs to have good estimates both of the stress concentration factor (SCF) at the weld toe and of the induced forces in the bolts. While it is often assumed that once the bolts are preloaded they are exempt from the fatigue phenomenon, the validity of such a simplified assumption requires justification. While an estimation of the axial load in the bolt is proposed here, there is also a further need to evaluate bolt bending in order to improve the fatigue resistance predictions obtained.

It is usual to assume that throughout the entire history of the loading potentially causing fatigue the joint behaves in a linear elastic manner so that the same value SCF is valid whatever the level of loading. With information on the loading cycle history and the derived stress range values in the part concerned, the fatigue resistance of the joint can be verified via a suitable S-N curve. Research carried out in France (Chabrolin & Ryan 1993) of weld toe fatigue cracking failures in flange joints of monotube sky-lift pylon structures with preloaded bolts led to fatigue recommendations including a parametric formula for determining the SCF for the design of flange joints which have been successfully used for a number of years now by industry and transport authorities. Cao & Bell (1993, 1996a) proposed a theoretical elastic model for flange bolted connections and the confrontation of their results with experimental and numerical data (Cao & Bell 1996b) shown good agreement.

In the models developed by Cao & Bell (1993,1996a) the radial deformations (expansion/contraction) of the tube due to axial load are neglected which led to an underestimation of the stress concentration factor at the tube to flange junction either when the flange is relatively thick or when bolt preloading rigidifies the joint. In this paper an improved model is proposed which takes the tube radial deformations into account. Results of this model are compared to those obtained by 3-D finite element analyses and in one case to experimental measurements.
2 ANALYTICAL MODEL FOR THE FLANGE JOINT

2.1 Model assumptions

Cao & Bell (1993,1996a) analytically studied the elastic behaviour of circular tube to flange joints using a model based on Khirchoff-Love plate theory for the flange combined with standard tubular shell theory. Since a fully continuous junction between the tube and the flange is considered, the geometry of the flange to tube weld is not modelled.

In a first model, Cao & Bell (1993) assumed the plate to be on a simple rigid support at the bolt perimeter and to be free at the outer edge of the circular flange plate. In a second model (Cao & Bell 1996a), the forces applied by bolts positioned in holes in the flange is simplified to the action of an equivalent “annular” axial spring positioned at the perimeter with the bolt positions. When the bolts are not preloaded the prying forces are assumed to act at the outer edge of the flange. Since the flange plate is modelled without bolt holes and the bolts are modelled as an equivalent spring applying only a uniformly distributed normal load to the flange. The absence of a bolt hole in the plate should compensate for any bolt bending.

Comparison made by Cao & Bell with experimental (Cao & Bell 1996b) and analytical results (Cao & Bell 1993, 1996a) for flange joints for tubes of diameter not exceeding 180mm show a good agreement.

However in their study, they neglected the radial deformation of the tube due to the tensile force in it. While adopting all other aspects of the Cao and Bell model, the authors of the present article propose to modify that model in order to take the effect of radial deformations into account. With the modified model, new expressions for the SCF at the weld toe and for bolt tensile force are obtained.

\[ \theta_i = \frac{1}{2\beta^2 D_t} \left[ 2\beta M'_E - F'_E \right] \]  
\[ w_i = \frac{1}{2\beta^3 D_t} \left[ -\beta M'_E + F'_E \right] - \frac{\nu T}{2\pi E t} \]

where

\[ \beta^4 = \frac{E t_i}{4R^2 D_t}, \quad D_t = \frac{E t_i^3}{12\left(1-v^2\right)} \]

and \(M'_E\) and \(F'_E\) are the local moment and shear force per unit of circumferential length applied by the flange to the tube end.

2.2 Effect of rotation and radial displacement at the tube/flange junction

For an axially loaded tube of mean radius \( R \) and wall thickness \( t_i \) and with the end conditions indicated in Figure 3, the following relations (Couchaux & Ryan 2009) for the local tube rotation and the radial displacement respectively at the flange to tube wall junction can be obtained:
The tension stress at the outer surface of the tube wall at a distance of \( x \) from the flange junction is given by:

\[
\sigma_x = \frac{T}{A} + \frac{6}{t_e} e^{-\beta x} \left[ \left( M'_e - \frac{F'_e}{E} \right) \sin(\beta x) + M'_e \cos(\beta x) \right]
\]

where \( A \) : Cross area of the tube

\[ (3) \]

If the axially loaded tube is perfectly fixed at the flange end, which corresponds to zero values for \( \theta_t \) and \( w_t \), the predicted value of the stress at the flange junction is 1,544 times the nominal axial stress, which is theoretically correct. It is noted that the model proposed by Cao & Bell (1993) predicts a stress equal to the nominal stress for this case.

2.3 Study of a simple case

Cao & Bell (1993), studied the case of a circular flange joint with the bolt which are infinitely rigid and prying action is neglected (see Figure 4). This example will be used in paragraph 2.4. The rotation of the flange (Cao & Bell 1993) at its junction with the tube is then given by:

\[
\theta_f = \frac{R}{D_f} \frac{a^2 - R^2}{a^2 - b^2} \left[ \frac{1}{1 + v} + \frac{b^2 - 1}{R^2 - 1 - v} \right] \left[ TK_s - \frac{M'_e + \frac{F'_e}{E} \theta_f}{2} \right] \cdot k_0
\]

where

\[ (4) \]

Furthermore the radial displacement of the flange at its junction with the tube can be obtained by (Cao & Bell 1993):

\[
w_f = -\frac{F'_e}{E} \frac{R}{t_f} \alpha
\]

where

\[ (5) \]

Considering the thickness of the flange, the continuity conditions for the tube and the flange are:

\[
\theta_f = \theta_t
\]

\[ (6) \]

\[
w_t = w_f + \frac{\theta_t t_f}{2}
\]

\[ (7) \]
Introducing (1), (2) and (5) in (7) we get the shear force per unit length at the junction tube-flange:

\[ F' = \frac{2\rho(\beta^{0.25} + \rho + 1)}{2\beta^{-0.25} + \rho + \frac{t_f}{t_f}h \cdot \frac{\alpha}{2\beta^{0.5} \rho}} \left( \frac{M'_E}{t_f} + \frac{\nu}{2\pi\alpha \left( \frac{R_{t_1}}{D_{t_1}} + \frac{R_{t_2}}{D_{t_2}} + \frac{t_f R_{t_1}}{4D_{t_1}^2} \right)} \right) T = \eta \frac{M'_E}{t_f} + \nu T \tag{8} \]

where \( \rho = \frac{t_f}{\sqrt{R_{t_1}}} \) and \( \beta' = 3(1 - \nu^2) \)

And introducing (1), (4) and (8) in (6) we obtain the bending moment per unit length at the junction tube flange:

\[ M' = \frac{T}{\Phi} \left[ k_3 + \nu \left( \frac{t_f D_f}{2\beta^{0.5} D_f} - \frac{t_f k_6}{2} \right) \right] = \frac{T}{\Phi} [k_3 + k_\nu] \tag{9} \]

where \( \Phi = \frac{\frac{t_f}{2\beta^{0.75} \rho - 0.5 \beta^{0.5} \eta}}{a^2 - R^2} + \frac{k_\nu(1 + \frac{\eta}{2})}{1 + 2 \nu} + \frac{a^2 - b^2}{a^2 - b^2} \left( \frac{1}{1 + \nu} + \frac{1}{R^2} \right) \)

In the model proposed by Cao & Bell (1993), \( k_3 \) is equal to zero.

![Figure 4: Loading and boundary conditions of the simple case](image)

2.4 Proposed model for complete flange connections

Cao & Bell (1996a) supposed that the deflection at the tube/flange junction is assumed to be zero. Considering the compatibility between flange and bolt and flange and tube they get the expression of the bolt force obtained here neglecting the presence of pretension:

\[ B = \frac{\left( \delta_{p1} - \delta_{p2} \right) - M_T \left( \delta_{M1} - \delta_{M2} \right)}{\delta_B + \left( \delta_{p1} - \delta_{p2} \right) - \left( \delta_{B1} - \delta_{B2} \right) - \left( M_T - M_M \right) \left( \delta_{M1} - \delta_{M2} \right)} \cdot T = \tan \varphi \cdot T \tag{10} \]

The expressions of \( \delta_{p1} - \delta_{p2}, \delta_{M1} - \delta_{M2} \) and \( \delta_{p1} - \delta_{p2} \) are obtained by Cao & Bell (1996a) and depend on the geometric characteristics of the flange. \( \delta_B \) is the flexibility of all bolts together in a joint for which we propose to take the following expression:

\[ \delta_B = \frac{L_b}{\nu b \cdot \frac{A_i}{n_b} \cdot E_A} \]

where \( A_i \): Cross section area,
\( L_b \): equivalent length calculated via EN1993-1-8 (2005),
\( n_b \): number of bolts.

The expressions for the moments per unit width \( M'_f \) and \( M'_p \) obtained via our model are different from those of Cao & Bell (1996a).

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$M_T^B$ is the total bending moment per unit width acting at the center of the flange thickness at the radius $R$ for a bolt force equal to unity (see figure 6).

$$M_T^B = \frac{k_3^B}{\Phi} \left[ 1 + \eta \right] + \frac{\psi f}{2}$$

(11)

where

$$k_3^B = k_3 + k_{3t}$$

$M_T^P$ is the total bending moment per unit width acting at the center of the flange thickness at the radius $R$ for a tensile force equal to 1 (neglecting the bolt force, see figure 7). Thus considering that $e = a$ in (9) we get:

$$M_T^P = \frac{k_3^P}{\Phi} \left[ 1 + \eta \right] + \frac{\psi f}{2}$$

(12)

where

$$k_3^P = k_3 \Big|_{e=a} + k_{3t}$$

(13)

The final expression of bending moment $M'_E$ per unit width at the tube flange junction is (Couchaux & Ryan 2009):

$$M'_E = \frac{(k_3^B - k_3^P) \tan \varphi + k_3^B}{\Phi} = mT$$

(14)

3 DESIGNS METHODS

3.1 Design procedure proposed by the authors

The SCF is the ratio of the normal stress due to local bending moment and nominal stress to the nominal stress. The normal stress at the weld toe can be calculated introducing (8) and (14) in (3). The weld toe is situated near the junction tube/flange and since the normal stress distribution is quasi linear along the tube axis, the bending moment can be obtained via the next expression:

$$M(c) \approx M'_E - c F'_{E}$$

(15)

Finally, the expression of the SCF at the weld toe becomes:

$$SCF = 1 + \frac{6 \cdot A}{t_f} \left[ m \left( 1 - \frac{c \eta}{t_f} \right) - c \psi \right]$$

(16)

Where $m$, $\psi$ and $\eta$ are calculated using equations (14) and (8). The ration between the bolt force and the tensile force is calculated via expressions (10), (11) and (12). However, the verification of the fatigue resistance of the bolt via Eurocode needs to take into account the stress due to bending moment.
3.2 SCF parametric formula

The following empirical formula for the weld toe SCF was developed (Chabrolin & Ryan 1993) from analyses of flange connections using plate and shell finite elements:

\[
SCF = 4.11 \left[ \frac{t_f}{6 \pi} \right]^{0.46} \left[ \frac{2\pi e}{n_p 243} \right]^{0.38} \left[ \frac{e - R}{56} \right]^{0.11} \left[ \frac{2R}{508} \right]^{0.21}
\]

(17)

4 NUMERICAL MODEL, VALIDATION

4.1 Numerical model

The numerical model was carried out with the Finite element code ANSYS V11.0. Cao & Bell (1996b) develop a similar model which was confronted with experimental results and good results were obtained. Connections were generated with three dimensional elements, which were hexahedral or tetrahedral bricks. An elastic linear law of Hooke is chosen for steel (E=210000MPa, v=0.3). The dimension of the bolt respect specification of EN14399-3 (2004). A constant cross-section is considered over the length including the thread considering the effective cross area of EN1993-1-8 (2005). Two types of contact elements are also used: a) Flexible contact elements between the flange and the bolt head and b) Rigid contact element between the flange and the fictive flange. An isotropic Coulomb friction law (\(\mu = 0.25\)) is used to reproduced sliding and sticking conditions between the flange and the bolt head. Friction is neglected between the two flanges because of the symmetry. In fact when the connections are subjected to a tensile force, it is possible to take into account the geometrical and loading symmetry. Hence just a quarter of the connections is represented. Tension is applied at the end of the tube and symmetry plane is created at the edge of the model. The presence of weld is neglected due to the fact that the SCF must be calculated neglecting influence of the weld as explained in paragraph 1. The finite element model has been validated via comparisons with experimental results from Cao & Bell (1993b).

4.2 Comparisons with analytical results

The results for two series of axially loaded tube connections are presented here. The first series is based on the blank flange tube connections tested by Cao & Bell (1996b) (named S-1 to S-4) for a tube of 114,3mm in diameter. The second series (see Table 1) represent typical circular tube ring flange connections of ski-lift installations pylons of a tube of 762mm in diameter.

<table>
<thead>
<tr>
<th>Connection</th>
<th>a (mm)</th>
<th>e (mm)</th>
<th>R (mm)</th>
<th>b (mm)</th>
<th>(t_f) (mm)</th>
<th>(t_t) (mm)</th>
<th>Bolt type</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-5</td>
<td>458,5</td>
<td>422,5</td>
<td>379,2</td>
<td>377,4</td>
<td>378</td>
<td>375</td>
<td>24M24</td>
</tr>
<tr>
<td>S-6</td>
<td></td>
<td></td>
<td>378</td>
<td></td>
<td>375</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>S-7</td>
<td></td>
<td></td>
<td>374,75</td>
<td>368,5</td>
<td>373</td>
<td>12,5</td>
<td>16</td>
</tr>
<tr>
<td>S-8</td>
<td></td>
<td></td>
<td>373</td>
<td></td>
<td>365</td>
<td>12,5</td>
<td></td>
</tr>
<tr>
<td>S-9</td>
<td>480</td>
<td></td>
<td>374,75</td>
<td>368,5</td>
<td>374,75</td>
<td>12,5</td>
<td></td>
</tr>
</tbody>
</table>

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The weld leg length is taken as equal to the tube wall thickness in all cases. The predicted evolution of the axial stress on the outer tube wall is presented in Figure 10 for connections S-1 and S-7. It is clear that the stress distributions obtained via analytical and numerical calculation are quite similar. Results of the analytical model of the authors are based on (3), (8) and (14) and are compared to those of Cao & Bell (1996a). For connection S-1, results are in agreement with the Cao & Bell (1996b) measurements.

![Figure 10. Evolution of the axial stress on the outer tube wall](image)

Results of the ratio of the bolt force to tensile force and the SCF at the weld toe are presented in Table 2.

<table>
<thead>
<tr>
<th>Connection and R/t</th>
<th>Ratio of Bolt force to tube tensile force : tan φ</th>
<th>SCF at the weld toe</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test result</td>
<td>3D FEM</td>
</tr>
<tr>
<td>S-1 11</td>
<td>1.35</td>
<td>1.37</td>
</tr>
<tr>
<td>S-2 145</td>
<td>1.45</td>
<td>1.48</td>
</tr>
<tr>
<td>S-3 127</td>
<td>1.27</td>
<td>1.28</td>
</tr>
<tr>
<td>S-4 163</td>
<td>1.63</td>
<td>1.54</td>
</tr>
<tr>
<td>S-5 105</td>
<td>-</td>
<td>2.05</td>
</tr>
<tr>
<td>S-6 63</td>
<td>-</td>
<td>2.13</td>
</tr>
<tr>
<td>S-7 30</td>
<td>-</td>
<td>1.87</td>
</tr>
<tr>
<td>S-8 23</td>
<td>-</td>
<td>1.76</td>
</tr>
<tr>
<td>S-9 30</td>
<td>-</td>
<td>1.64</td>
</tr>
</tbody>
</table>

In general there is quite good agreement between all approaches for all types of tubes. The Cao & Bell values are, as expected, lower than those obtained from the improved model by the present authors. Compared to the FEM results the author’s formulation always overestimates the SCF values while this is not the case for the Cao & Bell predictions. The accuracy of the Cao & Bell model improves as the ratio $R/t$, for the tube decreases.

The empirical SCF formula tends to overestimate the SCF values for the smaller tube connections and underestimate it for the larger tube connections.

5 SUMMARY

In this paper an extension of the analytical model of Cao & Bell (1996a) is proposed and used to determine the stress concentration factor at the weld-toe of tube to flange plate welded connections and the bolt force on the bolt.

A numerical model is developed using 3D brick and contact elements and the results are compared to analytical and experimental ones. The indications are that for an analytical model the effect of the
tube radial displacement due to tensile force should not be neglected because it tends to lead to underestimating the SCF value at the weld toe. However the relation obtained via the analytical model proposed by the authors is relatively complex and simplifications may be required, possibly in the form of an improved empirical formula. Further study is required especially to clarify the influence of bolt preloading but also of initial fabrication imperfections of the flange due to the welding with the tube.

REFERENCE


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